Lazy Acquisition of Place Knowledge

Pat Langley[◇] (Langley@cs.stanford.edu)
Institute for the Study of Learning and Expertise
2164 Staunton Court, Palo Alto, CA 94306

Karl Pfleger (KPfleger@hpp.stanford.edu)
Knowledge Systems Laboratory, Computer Science Dept.
Stanford University, Stanford, CA 94305

Mehran Sahami (sahami@cs.stanford.edu)
Robotics Laboratory, Computer Science Department
Stanford University, Stanford, CA 94305

Abstract

In this paper we define the task of place learning and describe one approach to this problem. Our framework represents distinct places as evidence grids, a probabilistic description of occupancy. Place recognition relies on nearest neighbor classification, augmented by a registration process to correct for translational differences between the two grids. The learning mechanism is lazy in that it involves the simple storage of inferred evidence grids. Experimental studies with physical and simulated robots suggest that this approach improves place recognition with experience, that it can handle significant sensor noise, that it benefits from improved quality in stored cases, and that it scales well to environments with many distinct places. Additional studies suggest that using historical information about the robot's path through the environment can actually reduce recognition accuracy. Previous researchers have studied evidence grids and place learning, but they have not combined these two powerful concepts, nor have they used systematic experimentation to evaluate their methods' abilities.

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[◇] Also affiliated with the Robotics Laboratory, Computer Science Department, Stanford University, Stanford, CA 94305

1. Introduction and Basic Concepts

A physical agent exists in an environment, and knowledge about that environment can aid its achievement of goals. One important type of environmental knowledge concerns the spatial arrangement of the agent's surroundings. For this reason, research on the representation, use, and acquisition of spatial knowledge has occupied an important role in the field of robotics. However, work on spatial learning has seldom made contact with the systematic experimental methodology that predominates in other areas of machine learning. In this paper, we consider a novel approach to this area that incorporates ideas from both of these disciplines.

We begin with some definitions of concepts and tasks that appear central to spatial reasoning. Consider a physical agent, say a robot, that is situated in the world. We can say that:

Definition 1 *The* POSITION *of an agent is a coordinate in 2D or 3D space.*

Position corresponds to ground truth, giving the actual location of the agent in some established coordinate system. We might also define the related concept of agent *orientation*, but here we will assume the agent has a 360 degree field of view, making this notion unnecessary.

A physical agent does not typically have direct access to knowledge of its position, but it does have indirect information.

Definition 2 *A* SENSOR READING *is a description of the environment around the agent's position that has been filtered through its sensors.*

The information in sensor readings may be imperfect in various ways. For example, it may be incomplete in that it describes only certain characteristics of the local environment, and it may be noisy in that sensor readings for the same position may produce different results at different times.

Nevertheless, the agent must find some way to use this information to make useful inferences. This suggests a natural task for a physical agent:

Definition 3 LOCALIZATION *involves determining the position of the agent in the environment from a set of sensor readings.*

Other tasks, such as navigating from position A to position B, are certainly possible. But an agent cannot begin to carry out such a task without first knowing A and without knowing when it has achieved B. Thus, localization seems more basic than navigation, and we will focus our attention on it here.

However, in many situations humans seem to care less about their exact position in space than about more abstract spatial regions. This suggests another, somewhat different, concept:

Definition 4 *A* PLACE *is a contiguous set of positions in 2D or 3D space.*

Robotics researchers have paid relatively little attention to the notion of place, but its central role in human spatial reasoning suggests it deserves a closer look. Naturally, this new concept lets us define an associated performance task by analogy with the localization task:

Definition 5 Place recognition *involves determining the place in which the agent currently resides from a set of sensor readings.*

At least in principle, the place recognition task seems more tractable (in terms of accuracy) than localization, in that it transforms a problem of numeric prediction into one involving discrete classification. One can also carry out localization within the context of a given place, but this in turn may be easier than global localization. Navigation between two places may also be simpler than navigation between two positions, as the former involves less precision than the latter.

Of course, reliance on places rather than positions also introduces a problem: one must specify some descriptions in memory that let the agent map sensor readings onto place names. One might attempt to enter such descriptions manually, but it seems desirable to automate this process, suggesting a final task:

Definition 6 Place learning *involves the induction of descriptions, from the sensor readings and place names for a set of training positions, that let the agent accurately recognize the places of novel positions.*

Note that this task formulation makes minimal demands on the teacher, who does not have to give the agent information about its actual positions. Rather, the agent collects its own sensor readings, and the teacher must only label each reading as an instance of one place or another. This formulation assumes supervised training data, but unsupervised versions, in which the agent decides on its own place names, are also possible. We will touch briefly on unsupervised place learning in Section 5, but we will focus on the supervised version in this paper.

In the pages that follow, we present one approach to dealing with knowledge about places. First we describe a representational formalism for storing place knowledge – evidence grids – and then examine a method for place recognition that operates on this representation, along with a simple learning process that acquires and refines knowledge of places. This approach to place learning is *lazy* rather than *eager*, in that the storage process involves only the retention of evidence grids, while generalization occurs at retrieval time, during the matching of new grids against those in memory. After describing this approach, we present some hypotheses about its behavior and some experimental tests of those hypotheses, then present some additional studies of the role of historical information in place recognition. Finally, we review related work on spatial learning and discuss some directions for future work.

One important difference between our approach and earlier robotics work on spatial knowledge lies in our incorporation of ideas from the experimental study of machine learning.[1] In particular, we view place recognition as a classification task and we view place learning as a supervised concept induction task. This suggests not only certain learning methods, but also the use of experimental methods prevalent in machine learning (Kibler & Langley, 1988) to evaluate our technique. However, the tasks of place recognition and place learning introduce some difficulties not usually present in such learning research, such as the pervasive presence of significant sensor noise, and our approach to the problem is designed with these issues in mind. These tasks also provide some

1. Other work in robotic learning (e.g., Atkeson, 1989; Moore, 1990) has fared much better in terms of experimental methodology, especially in the use of well-defined performance tasks.

information not available for most classification problems, such as historical context about previous places, which we consider later in the paper.

2. Representation, Use, and Acquisition of Place Knowledge

With the above definitions in hand, we can examine one approach to learning place knowledge. However, before we address the acquisition process, we should first consider the manner in which we represent knowledge about places and the performance element that takes advantage of that spatial knowledge base.

2.1 The Evidence Grid Representation

Robotics researchers have explored a variety of formalisms for representing spatial knowledge. One approach relies on geometric primitives to describe the edges or surfaces of obstacles in the environment. For example, one can use a set of lines to approximate the walls of an office and the furniture it contains. Such representations are precise, but Schiele and Crowley (1994) note that they can be difficult to use when sensors are noisy.

Another common scheme involves dividing the environment into a rectangular grid of mutually exclusive cells, each corresponding to a distinct position in space. In this framework, each cell is specified as either occupied (containing an obstacle) or open (containing none). This approach is well suited to navigation tasks in which one already knows the structure of the environment (i.e., which cells are occupied) and the position of the agent within the grid. However, this scheme is not designed to handle the uncertainty that arises when the position is unknown or when the agent has yet to learn the structure of the environment.

An alternative framework uses the *evidence grid* (Elfes, 1989; Moravec & Blackwell, 1992), a data structure that is specifically designed to tolerate uncertainty. In this approach, each cell C has an associated probability that C is occupied by some tangible object that would block the agent's path if it tried to move through the cell. These probabilities range from near zero (nearly certain a cell is open) to near one (nearly certain a cell is occupied), with the middle corresponding to cells for which little information is available (e.g., behind a wall or inside an object). We will adopt this framework in the current paper.

Figure 1 shows the position of an agent in a room within a larger office environment, similar in structure to an actual area at Stanford University. Figure 2 depicts evidence grids constructed from simulated sensor readings taken from positions (a) in the top left room and (b) in the lower left room from the same orientation. Open regions within the agent's view have low probability of being occupied (lighter shades) and that edges of obstacles and walls within view have high probability (darker shades). However, areas that are occluded, such as those behind obstacles and walls, have probabilities around $\frac{1}{2}$ (empty regions), since the agent's sensors provide no information about them. Of course, the agent can construct a more complete evidence grid by moving around the environment to collect sensor readings from different viewpoints.

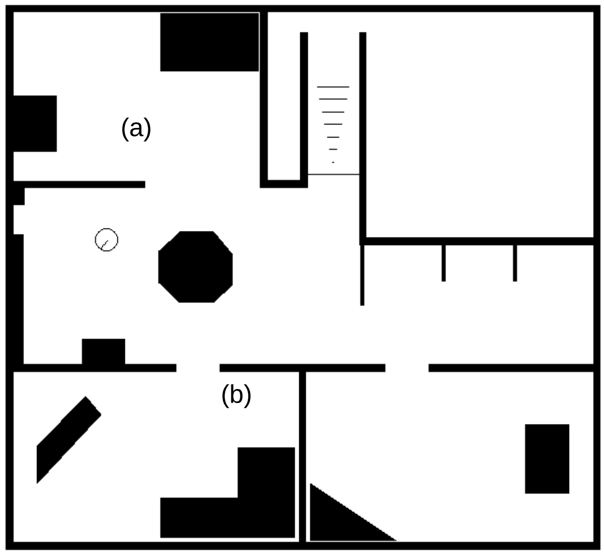

Figure 1. A simulated office environment with a number of distinct places.

Previous work with evidence grids (Elfes, 1989; Moravec & Blackwell, 1992) has emphasized their use in representing single rooms over a relatively short period. However, they also have potential for handling large-scale spatial knowledge over longer time spans. An agent could store its knowledge about an entire building or even a city in a single, large evidence grid. But this scheme seems impractical due to the difficulties inherent in integrating information from distant regions into a single map.

A more tractable approach to representing large-scale spatial knowledge, which we take here, involves storage of separate evidence grids for each distinct place. For example, one might use a different grid to encode each room in a building. This knowledge can be augmented by geometric relations among places, which would support navigation planning, but we will not address that aspect here. The retention of place descriptions in memory has much in common with the storage of a *case library* in work on case-based reasoning (Aamodt & Plaza, 1994; Kolodner, 1993). In both frameworks, the stored items represent alternative situations in which the agent can find itself, and which suggest different inferences.

2.2 Lazy Recognition of Places

Now that we have described the nature of evidence grids, we can examine their use in place recognition. Let us assume the agent has a stored place library, with each place described as an $R \times C$ evidence grid with an associated place name. Our approach to place recognition relies on a three-step process that, like other lazy methods, carries out much of the induction at performance time.

Figure 2. Evidence grids generated from simulated sonar readings for (a) the top left room in Figure 1 and (b) the lower left room in the same figure.

First, the agent constructs a temporary or short-term evidence grid for its current position from a set of sensor readings. This involves transforming each sensor reading into a probability of occupancy for each cell. Following Moravec and Blackwell (1992), we assume a sensor model that specifies this mapping. The result is a temporary evidence grid, based on the sensor reading, that characterizes the region in the vicinity of the agent. The agent may repeat this process a number of times, in each case incorporating the result into the temporary grid using a Bayesian updating scheme. We will not describe this updating process in depth, but readers can find details in Elfes (1989) and in Moravec and Blackwell (1992).

Next, the agent matches the short-term evidence grid against each of the grids stored in the place library. The evaluation function used in this comparison process measures the degree of match between two grids. Specifically, if $S_{r,c}$ is the probability associated with the rth of R rows and the cth of C columns for the short-term grid, and if $L_{r,c}$ is the analogous probability for the stored, long-term grid, then

$$M = \sum_{r}^{R} \sum_{c}^{C} F(S_{r,c}, L_{r,c})$$

computes the similarity between the short-term and stored grids. One can instantiate the function F in many ways, provided they satisfy certain properties: two cells should be treated as similar if they are confident in the same direction, as dissimilar if they are confident in opposite directions, and generally ignored if either is uncertain.

Moravec and Blackwell (1992) implement this cell-to-cell component of grid similarity as the expression

$$F(S_{r,c}, L_{r,c}) = log_2 \left[S_{r,c} L_{r,c} + (1 - S_{r,c})(1 - L_{r,c}) \right] + 1 \quad ,$$

which varies from one (a perfect match) to negative infinity (the worst possible match). Reflection suggests that this scheme might give very low match scores to reasonably similar grids if even a few cells are confident in opposite directions. For this reason, we decided to use an alternative metric:

$$F(S_{r,c}, L_{r,c}) = \begin{cases} 1 & \text{if } S_{r,c} > \frac{2}{3} \text{ and } L_{r,c} > \frac{2}{3} \\ 1 & \text{if } S_{r,c} < \frac{1}{3} \text{ and } L_{r,c} < \frac{1}{3} \\ -1 & \text{if } S_{r,c} > \frac{2}{3} \text{ and } L_{r,c} < \frac{1}{3} \\ -1 & \text{if } S_{r,c} < \frac{1}{3} \text{ and } L_{r,c} > \frac{2}{3} \\ 0 & \text{otherwise} . \end{cases}$$

We felt this measure would be less sensitive to situations in which disagreements arise between cells having high certainty, thus eliminating the problem predicted for the Moravec/Blackwell measure. There is nothing special about the choice of $\frac{1}{3}$ and $\frac{2}{3}$ as thresholds, as few cells have probabilities near them; the important point is to divide experience into three qualitative states.

The above metrics assume that the stored and temporary grids are described in the same coordinate system. One can plausibly assume the presence of a reasonably accurate compass to determine the relative rotations, but possible differences in translation requires some form of registration that coerces the temporary evidence grid into the same coordinate system as the stored place. To this end, our system carries out an exhaustive search using operators that modify the position by one grid row or column, evaluating each alternative using the metric M defined above.[2] The system selects the translation that gives the highest M score; the resulting registered grid localizes the agent with respect to that grid. If the compass is not accurate, one can extend this approach to correct for small offsets in rotation.

Finally, the agent compares the match scores for the various registered grids and selects the best of these competitors. This strategy provides both the place name associated with the selected evidence grid and the estimated position within that place description. Because adjacent evidence grids may cover overlapping regions, this scheme has some potential for misclassifying a place based on its outlying rather than its central cells. An alternative strategy would let the agent associate distinct place names with different cells in the same stored grid, then predict the name specified for the cell nearest to the estimated position. However, this issue has not been a problem in our studies to date, so our current system relies on the simpler classification strategy.

As we noted earlier, this approach has much in common with other methods for lazy recognition. Here the evidence grids in the place library correspond to stored experiences, whereas the short-term grid maps on to a test case for which one wants to make a prediction. The match function corresponds to the similarity metric that determines the nearness of the test case to each stored case in an $R \times C$ dimensional space, and the final classification step is similar to that used in the nearest neighbor method, perhaps the simplest lazy technique. The fact that each evidence grid is a probabilistic summary, computed from a set of sensor readings, differs from the prototypical lazy approach, but some systems partially generalize from experience at storage time. A more intriguing difference concerns the registration process. Many lazy systems incorporate some *adaptation*

2. When translation causes two grids to overlap on only $R' \times C'$ cells, the metric uses only these cells in its summation. This creates a bias toward stored grids that share more cells with the temporary grid, which seems reasonable, but it does not actively punish a stored grid for having only partial overlap. Although the current registration algorithm is exhaustive and thus computationally expensive, Alan Schultz (personal communication, 1995) reports encouraging results using a more efficient registration algorithm using genetic search with a similar match function.

method (Aamodt & Plaza, 1994; Leake, 1994), but usually this occurs after retrieval, whereas here adaptation (registration) takes place during the evaluation (match) process itself.

2.3 Lazy Learning of Place Knowledge

Now let us consider an approach to learning place knowledge that is stored as evidence grids. We would like an incremental process, since the agent encounters its environment sequentially. However, we are not concerned here with the task of effectively exploring an unknown world, so we will assume that the agent is led to a position, given time to observe its surroundings, given a place name, led to another position, and so forth.

Given our commitment to a place library and to a method for place recognition described above, we naturally assume a lazy learning scheme. In particular, at each position to which it is led, the agent constructs a short-term evidence grid S using the method described above. The system then simply adds the new grid to the place library, along with specified place name. The same place name may be associated with multiple evidence grids, but this seems appropriate if they produce different sensor readings.

At first glance, this approach to place learning sounds guaranteed to work, in that one simply stores a description for each place, after which recognition will be perfect. However, this view ignores the central feature of the task – uncertainty. Even with noise-free sensors, the same place typically looks different from different positions, if only because different regions are occluded. Moreover, standard robotic sensors such as sonar are notoriously noisy, and will produce different sensor readings, and thus different evidence grids, even when repeated from the same position.

In addition, the dimensionality of the resulting space is high, with one attribute for each cell in the $R \times C$ evidence grid. Among others, Aha (1990) and Langley and Sage (in press) have shown that the learning rate of lazy methods like nearest neighbor can be drastically slowed by the presence of irrelevant attributes. Since typical rooms contain large open areas, it seems plausible that the cells that describe such areas will make place learning difficult. Thus, the adequacy of this approach remains an open question that is best answered by experiment.

2.4 Lazy vs. Eager Approaches to Place Learning

In addition to the lazy approach to place learning we have described above, we also considered eager methods that incorporated the evidence grid representation. However, the latter proved problematic in that nearly all eager learning methods, including connectionist and decision-tree techniques, assume a fixed set of attributes or features,[3] whereas evidence grids can have different sizes and thus different numbers of cells.

We considered one response to this problem that would coerce the training grids into a single size by padding extra cells with $\frac{1}{2}$ probabilities, thus ensuring a fixed feature set. However, this scheme would not guarantee that still larger grids would not occur in the test set, which would make it

3. The main exceptions are methods for inductive logic programming (Lavrac & Dzeroski, 1993), but their first-order representations hardly seem suitable for dealing with evidence grids.

difficult to apply the learned knowledge. Moreover, this strategy would increase the number of features in an already high-dimensional space, exacerbating the effect of irrelevant attributes.

The need to register evidence grids also poses difficulties for purely eager methods. Although one can imagine an eager learning scheme inducing higher-order, translation-invariant features, this would seem to require many more training grids than our lazy approach, since it would need to find regularities over many translated grids of the same places. An alternative approach would coerce all training grids for a given place into a single coordinate system, then use an eager method to learn place descriptions in terms of those coordinates. However, the resulting system would still have a strong lazy component, in that test grids would still require registration.

In summary, the evidence grid framework lends itself nicely to a lazy approach to place recognition and learning, but raises significant problems for eager techniques. Of course, this does not imply that eager approaches to place learning are impossible, as we will find in Section 4 when we discuss related work on this task. But for now our focus will remain on the lazy framework outlined above.

3. Experimental Studies of Place Learning

In Section 1 we formulated the place learning task in terms similar to those used to describe other induction problems. Thus, we can use the experimental methods developed for machine learning to evaluate the robustness of our framework. In this section we present a number of hypotheses about the system's behavior, followed by experimental tests of those hypotheses, most of which we have reported previously (Langley & Pfleger, 1995). Our primary measure of performance was recognition accuracy for places in a test set of evidence grids that differ from those in the training set.

3.1 The Experimental Setting

The experiments we designed to evaluate the abilities of our approach relied on both a physical robot – a Nomad 200 with a 16-sensor sonar ring – and a high-fidelity simulation of this machine. The physical environment was a suite of offices and common areas at Stanford University, and the simulated environment was an idealized layout of a similar suite, depicted earlier in Figure 1. We used the physical Nomad to ensure realism in our results, while the simulation gave us experimental control over device parameters not possible with the actual robot.

We generated each training or test case by placing the physical or simulated robot in a position, collecting readings from the sonar ring to construct an initial evidence grid (Elfes, 1989), rotating and/or moving the robot (as described below), collecting new sonar readings and updating the evidence grid, and repeating this process many times. For the simulated robot, we generated six different grids for each of six distinct places,[4] giving 36 total evidence grids. For the physical robot, we produced only three grids for each place (because the process took longer), giving 18 total grids.

4. These places corresponded to the lower left, lower right, middle right, and upper left rooms in Figure 1, and to the areas to the left and right of the octagonal table in the figure.

The Nomad simulator incorporates a number parameters that affect the quality of sonar information. For example, the `error` parameter controls random variation in the distance returned by the sonar sensors, `critical` controls the angle of incidence at which specular reflection occurs (giving distances farther than the actual ones), and `halfcone` controls the angular width of each sonar signal. Unless otherwise specified, we set `error` to 0.15, which was our best estimate of the error encountered by the physical robot, and we left all other parameters at their default values, which produce a 25 degree field of view for each sensor and specular reflection at angles of incidence with the sensed surface of 30 degrees or less.

For each experimental condition with the simulated environment, we ran the learning system 400 times with different random partitions of the evidence grids into 33 training and three test cases, randomly ordering the storage of training cases. For the physical environment, we randomly partitioned the grids into 17 training cases (with randomized orders) and one test case, again averaging over 400 runs for each condition.

3.2 Improving Place Recognition with Experience

Following Kibler and Langley (1988), we can divide the factors that affect the learner's behavior into two broad types, those involving characteristics of the environment and those involving features of the learner. The most basic environmental characteristic is the number of training cases available. Naturally, we hypothesized that the accuracy of place recognition would improve as the agent encounters more positions. However, the literature sometimes reports actual decreases in performance, so we needed to explicitly test this expectation.

As we report elsewhere (Langley & Pfleger, 1995), our first study used the physical Nomad robot to generate 18 evidence grids was based on 45 sonar readings, taken from a single position but with successive rotations incremented by one degree (though still merged into a stored grid with a single orientation). As expected, the system's ability to recognize places gradually increases as it observes and stores more training cases. However, the learning task is not trivial, in that multiple cases for each place are needed to achieve even 70% accuracy.

Inspection of the inferred structures reveals that, from certain views, the registered evidence grids for two different places occasionally appear more similar than the grids for two different positions within the same place. This should not be surprising, given the noise inherent in sonar sensors and given that objects can occlude portions of a place from some positions. Table 1 shows the actual confusions that occur, on average, for the physical robot after 17 training cases; because we included three grids for each place and used leave-one-out to estimate error rates, all entries are divisible by three. The table reveals that most errors involve the misclassification of place (e), which is confused with places (a) and (c), and the mislabeling of (d), which is classified as (c).

We also found that runs with the simulated robot produce a learning curve with a very similar shape to that for the physical device, but that the rate of learning is somewhat higher. Table 1 also shows the averaged confusion matrix for this experimental condition after the learner has seen 35 training cases; here we used six grids for each place, so each entry is divisible by six. Although a few errors still occur, the performance component generally assigns the correct place name to the

Table 1. Confusion matrices with probabilities of labeling for the six places used in the experiments. Rows indicate the correct place names, whereas columns show the predicted place after training.

Physical Robot	(a)	(b)	(c)	(d)	(e)	(f)
(a)	1	0	0	0	0	0
(b)	0	1	0	0	0	0
(c)	0	0	1	0	0	0
(d)	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0
(e)	$\frac{2}{3}$	0	$\frac{1}{3}$	0	0	0
(f)	0	0	0	0	0	1

Simulated Robot	(a)	(b)	(c)	(d)	(e)	(f)
(a)	$\frac{5}{6}$	0	$\frac{1}{6}$	0	0	0
(b)	0	1	0	0	0	0
(c)	0	0	1	0	0	0
(d)	0	0	$\frac{1}{6}$	$\frac{5}{6}$	0	0
(e)	0	0	$\frac{1}{6}$	0	$\frac{5}{6}$	0
(f)	0	0	0	0	0	1

test cases. The same experiment revealed that the method's ability to identify the precise position of the robot within a given place, based on the registration process, also improved with training.

3.3 Sensor Noise and Grid Quality

The amount of sensor noise constitutes a more interesting environmental factor. We might expect increased noise to reduce the asymptotic accuracy, as it can produce confusions between similar places, but it should have an even greater effect on the rate of place learning, in that it should increase the number of training cases needed to reach a given accuracy level. Nevertheless, we hoped that the probabilistic nature of evidence grids would let our approach degrade gracefully with increasing amounts of sensor noise.

Figure 3 shows two evidence grids constructed from simulated sonar signals collected from the same position and orientation within the lower left room in Figure 1. For grid (a), we set the simulator's `error` parameter to zero, so that there was no sensor noise. For grid (b), we set this parameter to 0.45, producing significant noise. The resulting evidence grids are similar but contain noticeable differences, suggesting that the basic inference method is robust but that sensor noise also has some effect.[5]

Fortunately, our reliance on evidence grids suggests a natural response to noisy sense data. Because each stored grid can be based on multiple sensor signals, we can attempt to improve the *quality* of these grids by increasing the number, or altering the arrangement, of the signals used to generate them. We hypothesized that place descriptions based on more sensor readings would be less affected by increases in sensor noise. Thus, we predicted an interaction between these two independent variables, specifically one that affects both learning rate and asymptotic accuracy.

5. The inferred "arms" in (b) appear to be artifacts of the grid updating scheme; the more generic effect of sensor noise is to create more ragged boundaries along the edges of objects.

Figure 3. Evidence grids generated from 45 simulated sonar readings for the lower left room in Figure 1 using (a) zero sensor noise and (b) a 0.45 noise setting.

To test this hypothesis, we used the Nomad simulator to produce four different levels of sensor noise, in which the `error` parameter was set to 0.0, 0.15, 0.30, and 0.45, respectively. We also attempted to vary the quality of the stored grids by using two different sensing strategies. In one, we based each evidence grid (both training and test cases) on 45 sonar readings collected from a single position but produced at orientations one degree apart, as used to generate the earlier results. In the other, we based each grid on 90 readings, produced by repeating this strategy in two nearby positions within the same room.

Figure 4 (a) shows the learning curves that result for the zero and 0.45 noise levels using the one-position sensing strategy, whereas Figure 4 (b) presents analogous results for the two-position strategy. (The results for the 0.15 and 0.3 settings fell between these extremes; we have omitted them for the sake of clarity.) The two-position scheme clearly fares better than the simpler strategy, and the curves generally agree with our predictions. The rate of learning for the two-position method is much higher than for the one-position method, even when no sensor noise is present. Also, the introduction of sensor noise clearly affects both strategies, but it alters only the learning rate for the more sophisticated scheme, while it actually appears to reduce the asymptotic recognition accuracy for the simpler one.

The general superiority of the two-position strategy is hardly surprising, in that its evidence grids are based on twice as many sonar readings. Ideally, we would prefer a sensing scheme that is robust with respect to noise but that requires no more sensing than the initial strategy. To this end, we explored a third method that takes three sonar readings at an initial position, with rotational increments of 7.5 degrees, moves a fixed amount along a straight line and takes another three readings in the same manner, then repeats this process until completing a total of 45 readings. The resulting evidence grid is based on sensing over the entire path, reducing the chance of occlusion and hopefully reducing the effect of sensor noise.

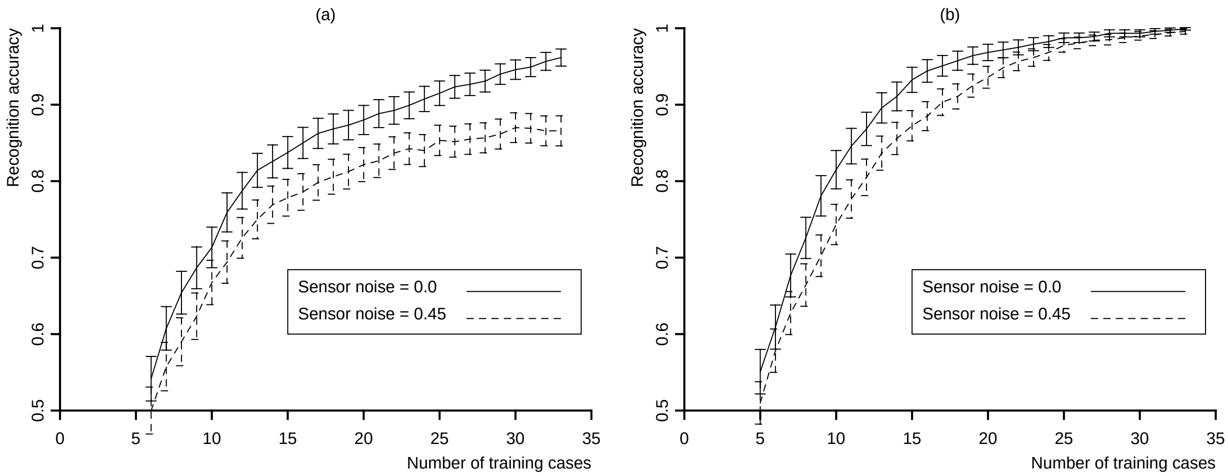

Figure 4. Learning curves for the lazy place learning system for two levels of sensor noise when evidence grids are based on (a) 45 readings from one position and (b) 90 readings from two nearby positions.

Figure 5 (a) presents learning curves for this sensing strategy on two of the simulated noise levels. For the noise-free situation, the behavior is nearly identical to that for the 90-reading strategy, even though grids are based on half as many sonar signals. However, sensor noise significantly degrades this strategy's behavior, though its accuracies remain well above those for the one-position method. Clearly, basing evidence grids on a number of distinct positions within a given place gives better results than basing them on one position, but increasing the number of readings also has desirable effects. It seems likely that more sophisticated sensing strategies, which sample readings in a more intelligent manner, would produce even better results.

3.4 Effect of the Similarity Metric

In Section 2.2 we described the similarity metric used to assign a short-term evidence grid to the stored grid that best matches it. This metric sums over the cells on which the two grids overlap, using a function F to measure the similarity of individual cells. We contrasted our implementation of F, which takes on the values 1, 0, and -1, with the implementation used by Moravec and Blackwell (1992), which ranges from one to negative infinity. We presented some intuitive arguments for preferring our formulation, but the question of which measure behaves better in practice is ultimately an empirical one.

Figure 5 (b) presents experimental results for the two similarity metrics, using training and test cases from six places based on 45 simulated sonar readings from one position. The learning curve for our version of the F function is similar to those we have seen earlier in the paper. In contrast, the curve for the Moravec and Blackwell metric reveals learning at a much slower rate, reaching only 39% accuracy after 33 training cases, as compared with 87% for our measure. These results do not imply that our approach is the only viable option, but they do show that the similarity measure can make a substantial difference in place recognition, and that our metric performs much better than one proposed alternative, at least on this task.

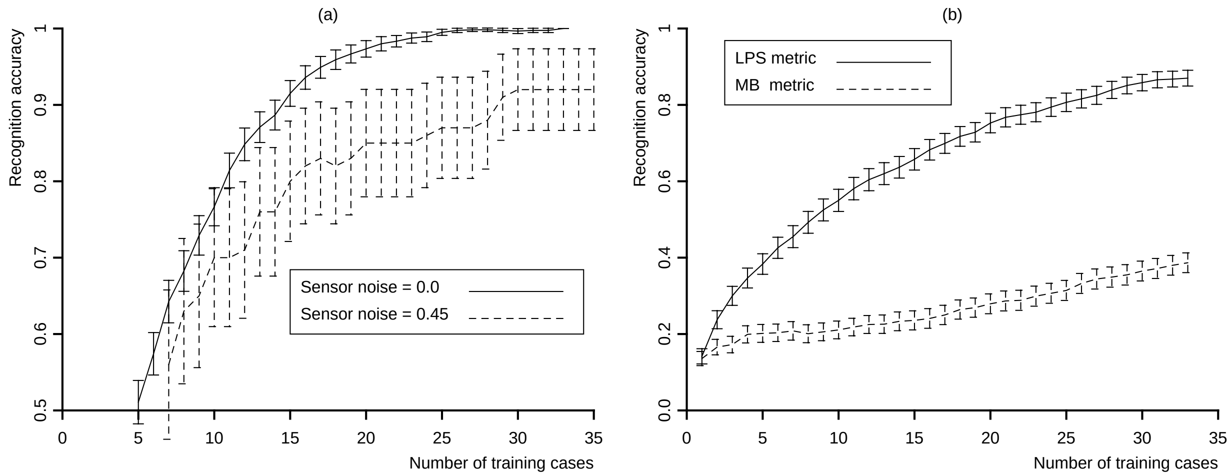

Figure 5. (a) Learning curves for two levels of sensor noise when evidence grids are based on 45 readings taken at equal intervals along a line between two positions. (b) Learning curves using the Langley/Pfleger/Sahami (LPS) similarity metric and the Moravec/Blackwell (MB) metric.

3.5 Number of Distinct Places

Some real-world environments contain many distinct places, making it desirable for a learning method to scale well as the number of places increases. We obtained preliminary results along these lines by examining our algorithm's behavior with different subsets of the places available in our environment. Figure 6 (a) shows the learning curves that result for two through six places, with each grid based on 45 simulated sonar readings from one position. Each reported accuracy is averaged over 400 runs for each possible subset of k out of six places, using 35 randomly selected training cases and one test case. Thus, when $k = 2$ we carried out $\binom{6}{2} \times 400 = 6000$ runs, and when $k = 3$ we carried out $\binom{6}{3} \times 400 = 8000$ runs. We have not reported confidence intervals here, since the accuracies are averages of averages.

Naturally, increasing the number of places decreases the speed of learning, but we can also examine the rate of this decrease. Note that the figure also shows where each learning curve crosses the level of 90 percent accuracy. These crossover points produce the *scaling* curve in Figure 6 (b), which maps the number of distinct places against the number of training cases needed to reach this accuracy level. This higher-order curve seems to be either linear or quadratic, but the analogous scaling curves for the two-position and straight-line sensing strategies, also shown, definitely appear linear. These results suggest that our approach requires, more or less, a fixed number of training cases per place, independent of the total number of places. This encourages us to believe that the method will scale well to domains that involve many more different places than the six we have examined, though ultimately we should test this prediction using larger environments.

3.6 Summary of Experimental Results

In this section, we reported on a number experiments designed to evaluate our lazy approach to the acquisition of place knowledge. We used a method common in research on machine learning, stating explicit hypotheses and running experiments designed to test them. In each case, we varied one or two independent variables and observed their effects on some performance measure.

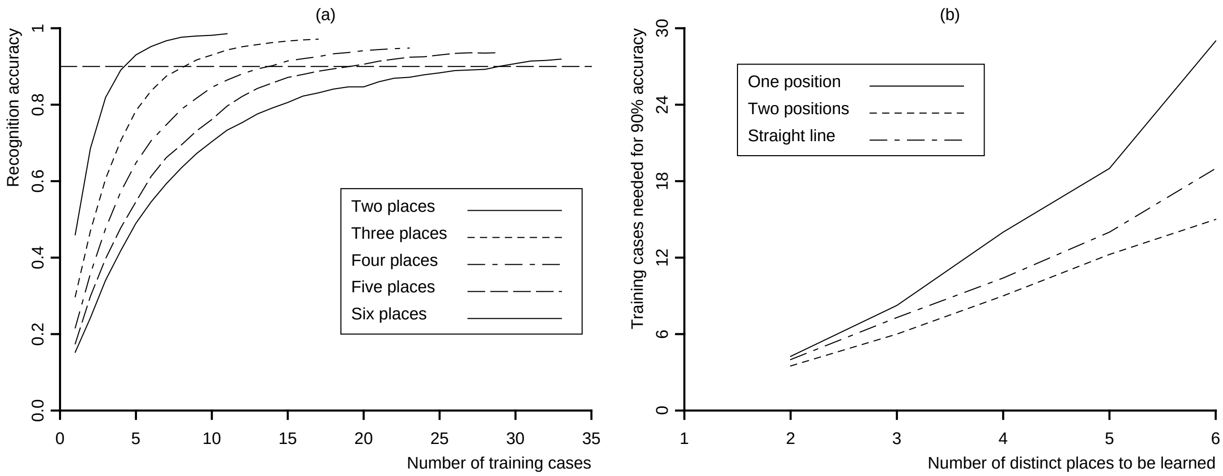

Figure 6. (a) Learning curves for different numbers of distinct places, based on 45 sonar readings from one position. (b) Scaling curves that map, for different sensing strategies, the number of training cases needed to achieve 90% accuracy as a function of the number of places.

The experiments revealed a number of encouraging behaviors. Our approach to place learning generally improves its recognition accuracy as it observes more training cases, with similar results occurring for both the physical and simulated robot. The learning rate slows in the presence of sensor noise, but one can mitigate this effect by increasing the quality of the inferred evidence grids. The rate of learning also slows with increasing numbers of places, but no more than expected in any multiclass learning situation. In addition, we found that our similarity metric performs significantly better than another metric proposed in the literature on this domain.

Clearly, there exist many other factors that could influence the behavior of our place-learning method. These include the resolution of the evidence grids and the distinctiveness of the places one must learn to distinguish. However, we will reserve these issues for future studies, as the current experiments have been sufficient to show that our approach is a promising one.

4. The Role of History in Place Learning

The above experiments dealt with place recognition in isolation, but this seems unrealistic for most robotics settings.[6] More often, a physical agent will have strong expectations about its current place based on knowledge about the place it has just left. Such historical information about the connections among places should be particularly useful for distinguishing between places that are otherwise similar.

One can encode knowledge of this sort in a topological map that takes the form of a Markov model. Each node in the map corresponds to a distinct place, while links indicate adjacency relations between pairs of places, along with the probability of moving from one place to a neighbor. Each node also specifies the prior probability that the agent is located there, lacking other information.

6. Such place recognition might occur when a robot is first turned on or when it reenters a known place during exploration, but these hardly seem typical.

In this section we provide an analysis of this framework, followed by initial experiments on the effects of previous place knowledge that moves beyond our earlier work.

4.1 Analysis of Historical Information

Suppose a robot has constructed an evidence grid E based on sensor information, and we would like to predict the current place C based not only on E but also on knowledge about the place B in which the robot was previously. Mathematically, we would like to compute $P(C|E,B)$, the probability that the robot is in current place C given previous place B and evidence grid E. As we show in the Appendix, under simple assumptions this term can be rewritten as

$$P(C|E,B) = \frac{P(C|E)P(B|C)}{\sum\limits_{C}[P(C|E)P(B|C)]} \quad .$$

However, to make this expression operational, we must define $P(B|C)$ and $P(C|E)$. We can expand the former to

$$P(B|C) = \frac{P(B)P(C|B)}{\sum_{B} P(B)P(C|B)} \quad ,$$

in which the terms $P(B)$ and $P(C|B)$ are known, provided we make Markov assumptions about the environment. We can expand the latter to

$$P(C|E) = \frac{\sum_{g} match(C_g, E)}{\sum_{D}\sum_{h} match(D_h, E)} \quad ,$$

where the summation in the numerator is over all stored grids g for place C, where those in the denominator are over all places D and all grids h for those places, and where $match(K_j, E)$ is the match score between the stored evidence grid j for place K and the sensory grid E. This expansion only approximates $P(C|E)$, but the accuracy of the approximation should increase with the number of grids stored for each place.

The above method takes the "proper" Bayesian approach of using a weighted vote over all candidate grids, but it differs from the best-match scheme we used in our earlier studies. We can obtain an analogous probabilistic version of the best-match method by defining the probability of a place $P(C)$ given a grid E, $P(C|E)$, as

$$P(C|E) = \frac{\max_{g} match(C_g, E)}{\sum_{D} \max_{h} match(D_h, E)} \quad .$$

In the absence of information about previous places, this best-match expression should give the same predictions as the method described in Section 2.

We use a simple algorithm that incorporates these expressions to make predictions about the current place C. First we initialize the distribution over previous places $P(B)$; in the studies reported below, we assumed a uniform distribution. Next, for each possible place C, we define the term

$$R(C) = \sum_{B}[P(C|E,B)P(B)]$$

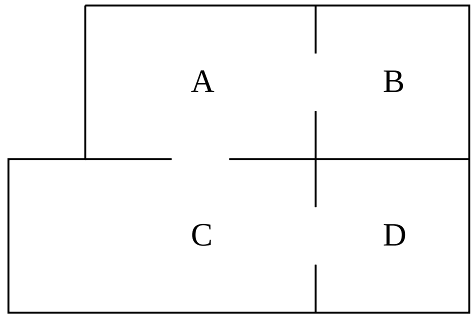

Figure 7. Simulated office environment with a four rooms, two of which (B and D) should be difficult to distinguish without historical path information.

using the expressions described above, which includes the match score for the current evidence grid and which marginalizes over all possible previous places. We then predict the place C with the highest $R(C)$ probability. Finally, we use the $R(C)$ value for each place as the prior probability for each previous place in the next round of reasoning, and we update $P(B|C)$ using this new prior. This updating process makes the Markovian assumption that knowledge of previous places on the current time step depends only on knowledge on the previous time step. This seems reasonable, as we are not concerned with how the agent has reached its current location, but only with the location itself.

4.2 Experimental Studies with Historical Information

We designed a simulated environment that would let us evaluate experimentally the influence of historical knowledge on place recognition and learning. Figure 7 shows the layout, which includes two distinctive rooms, A and C, and two identical rooms, B and D. We hypothesized that both the averaging and best-match schemes would fare better in this domain when historical information was available than when it was absent. To provide an upper bound on the effect of history, we decided to include a condition in which the programmer told the classifier the correct label for the previous place, thus providing perfect knowledge of the previous path.[7]

We used the Nomad simulator to generate six grids for each place in this environment, giving a total of 24 grids, which we split these into six disjoint sets, each containing one grid from each place. From these sets we generated six separate training/test splits by taking the union of five sets for training and reserving the remaining set for testing. For each group of training grids, we ran the learning system on ten distinct paths through the space; these were randomly selected from the possible paths of length seven or nine for the environment in Figure 7 in which each place was visited at least once, and excluding paths in which the robot remained in the same place across a time step. In addition to storing each training grid and its place name in memory, the learner

7. We also included a condition in which we gave the system the correct label for only its starting place. Although we do not report the precise results here, this condition always gave accuracies between those for probabilistic and certain historical knowledge.

Table 2. Place recognition accuracies (with standard errors in parentheses) with and without knowledge of previous places, for both averaging and best-match strategies, in environments with rooms that have open and closed doors.

Type of previous place knowledge	Open Doors		Closed Doors	
	Averaging	Best Match	Averaging	Best Match
None	0.611 (0.003)	0.878 (0.001)	0.908 (0.000)	0.908 (0.000)
Probabilistic	0.550 (0.002)	0.558 (0.007)	0.553 (0.004)	0.608 (0.004)
Certain	0.847 (0.001)	0.969 (0.000)	1.000 (0.000)	0.969 (0.000)

used the training paths to estimate $P(C|B)$ for each combination of places. We measured the classification accuracy on the test set for each path and for each split, then averaged the scores.

The leftmost columns of Table 2 show the results, a number of which are unexpected. First, in the absence of historical information, the best-match scheme is 27% more accurate than the Bayesian averaging method. Even more surprising, the use of probabilistic historical knowledge, inferred from previous grids along the path, actually decreases the accuracy for both the best-match and averaging strategies. The availability of certain knowledge about the previous place, provided by the programmer, helps for both the best-match and the averaging methods.

Inspection of the evidence grids inferred for this environment gives a partial explanation of the behavior. Because the sonar signals reach into adjacent rooms, the grids for B and D can appear quite different, even though the rooms themselves are identical. In other words, sonar information about adjacent rooms provides context that offsets the advantage of historical knowledge about path traversal. Moreover, this situation can lead to high variation in the grids generated for different positions within a given place, as the adjacent room will only be visible from some viewpoints.

These observations explain why knowledge of previous places provides little aid in this environment, but not why the probabilistic version actually hurts accuracy. Our hypothesis for this effect is that even occasional errors in place recognition early in the robot's path propagate to decisions later in the path, causing errors that do not occur when using only sensory information. This explanation is supported by the fact that, when provided by the programmer, historical information increased rather than reduced accuracy.

Because rooms were more distinguishable than we had expected, we decided to repeat the study with a similar environment that had closed doors. This configuration should remove any sensory context and thus make rooms B and D more difficult to discriminate. Table 2 also shows the results of this experimental condition. The main difference from the previous world is that the averaging and best-match methods give identical results in the absence of historical knowledge, which can be explained by the reduction in variability among the stored grids for each place. However, the use of probabilistic knowledge about previous places still reduces accuracy for both strategies over simple use of the current evidence grid.[8] As before, programmer-provided historical knowledge improves

8. Because places A and C occur more often than B and D in this environment, the expected accuracy is 82% for the closed-door world in the absence of historical knowledge, but this is considerably lower than the 91% observed.

both strategies, again suggesting that classification errors early in the path are responsible for the decrements with the probabilistic scheme.

These results raise serious questions about the usefulness of topological knowledge to constrain the process of place recognition. At least for the techniques and environments explored to date, the simple use of sensory information, transformed into an evidence grid and combined with a lazy classification method, appears to be the method of choice. Historical information can aid place recognition, but only when this knowledge is accurate enough to keep from introducing new errors.

5. Related Work on Learning Spatial Knowledge

Our research on the acquisition of spatial knowledge is certainly not the first in this area. Clearly, our work owes a strong intellectual debt to Elfes (1989), Moravec and Blackwell (1992), and other developers of the evidence grid framework. Our basic representation and our performance system directly employ techniques developed by these researchers. However, most research in this framework has focused on the construction of a single global map, rather than a collection of evidence grids for distinct places. Although such approaches clearly acquire spatial knowledge, they do not involve induction in the sense of using training instances to improve performance on novel test cases, whereas our work on place learning fits easily into this paradigm. Thrun (1993) has used reinforcement learning to improve sensor interpretation for evidence-grid construction, but his goal was to construct a global map. Mahadevan (1992) describes a method that forms generalizations expressed as evidence grids, but his aim was to learn not places but action models.

Nevertheless, some researchers outside the evidence grid formalism have studied place learning. For example, Yamauchi and Beer (1994) describe Elden, a system that represents places in terms of means and variances of direct sensor readings, rather than inferred grid occupancies. Their place descriptions also include features for the robot's position as estimated through dead reckoning and connections to recently visited places. Place recognition involves passing each attribute's value through Gaussian functions associated with each place, then selecting the competitor with the highest sum. Learning consists of updating the means and variances for recognized places, creating new places when no existing ones match well enough, and adding predictive connections between places. Yamauchi and Beer's reliance on a Gaussian distance metric makes their method similar to our approach, though Elden differs in the eager nature of its instance-averaging process, its use of raw sensor data, and the unsupervised nature of the learning processes.[9]

Lin, Hanson, and Judd (1994) have taken a similar approach to representing and using spatial knowledge. Their system also describes places[10] as means and variances of sonar readings and uses a Gaussian metric to determine the degree of match against the current sensor signals. However, their learning mechanisms, which are best described as eager rather than lazy, include not only the

Inspection of the confusion matrix shows that, as expected, place D was misclassified half of the time, but that B was always correctly classified. This appeared to result from differences in the evidence grids due to specular reflection, which occurred more in B than in D due to chance.

9. Yamauchi and Beer's system also incorporates an evidence grid representation, but it constructs a global map and uses this map for correcting errors in dead reckoning rather than for place recognition.

10. Lin et al. refer to their descriptions as *landmarks*. However, this term usually indicates a feature of the environment used to distinguish among different places, rather than to places themselves.

creation and updating of place descriptions, but also a reinforcement process designed to improve estimates of the robot's location. This latter technique can lead the learner to add a new place or remove an existing one if these actions reduce errors in location estimates.

Kuipers and Byun's (1988) NX system also operates on direct sensory readings, but it stores only places that are distinctive in terms of optimizing certain measures. For example, NX defines the central point in a hallway corner as being symmetrical and being equidistant from the walls, in addition to containing information about the angles and distances to obstacles. The system also describes edges, which connect distinctive places, in terms of length, width, and similar characteristics. Whenever NX encounters a local optimum L on one of its measures, it compares the sensor readings to each known place P stored in memory; if the descriptions for L and P are similar, and if their locations are metrically or topologically close, the system classifies L as the known place P. Otherwise, NX creates a new place based on L's description and stores this in memory, along with its edge connections to other places. This approach to place learning is lazy, like our own, in that little processing takes place at storage time. Mataric (1991) describes a similar scheme, though the details of place creation are different.

In methodological terms, Kortencamp and Weymouth's (1994) work is perhaps the most similar to our own. Their approach emphasizes *gateways* such as doors that connect two regions, but their system represents these locations using a grid structure and they evaluate its behavior in terms of recognition accuracy. However, their scheme uses hand-coded descriptions for a few gateway types to recognize candidate places and create new ones, rather than actual supervised training data, and they compare a number of different recognition strategies, including one that combines evidence from sonar and visual sensors.

On another dimension, our approach is most similar to Yeap's (1988) work on spatial reasoning. His framework also posits the storage of distinct places, the descriptions of which are not direct sensory readings but inferred summaries. However, his "absolute space representation" does not take the form of evidence grids but rather consists of a connected sequence of line segments that, except for occasional openings, enclose an area. Yeap does not describe a performance element that uses these descriptions in place recognition but, as in our own framework, learning involves the simple storage of the inferred place descriptions, which suggests the use of a lazy method.

6. Concluding Remarks

Although our experimental studies of place learning have revealed some insight into our approach, clearly more work remains to be done. The most immediate extension would replace the current supervised learning method with an unsupervised one. Such a system must identify distinctive places on its own, as it cannot rely on a tutor for this information. To this end, we plan to employ a technique similar to that used by Anderson and Matessa (1992) for classification and by Yamauchi and Beer (1994) for place recognition, but adapted to operate on evidence grids rather than direct sensor descriptions. As the agent moves through the environment, it would regularly stop and construct a short-term evidence grid, merging this with the previous place description if the match is high enough and using the short-term grid as the basis for a new place otherwise.[11] Discontinuities caused by passage through doors and past obstacles should be enough to identify distinguishable places.

Most methods for place learning, including those discussed above, also construct topological maps that connect different places. Clearly, this is another important direction in which to extend our approach. We expect that storing rough estimates of the direction of movement between one place and its successor will be sufficient for many navigation tasks. Upon executing a navigation plan, the agent would still need to register its location upon entering each place along the path, but expectations about the next place and its rough translation should greatly simplify the registration process. Although our preliminary results with the use of historical information to reduce confusion among similar places were not entirely positive, it seems likely that including additional information, such as direction of movement, would improve the situation. Such context should also reduce the computational complexity of place recognition by provided expected positions and orientations from which to hill climb toward a good registration.

In future work, we also hope to develop methods for detecting distinctive features in evidence grids that would simplify the place recognition process. We envision such features as being configurations of grid cells with large differences in their probabilities, such as might occur along a wall or at a door. The recognition mechanism would use the presence of these features as cues during retrieval of candidate places and during registration, and the learning process would use the features to index places in memory. Such learned features could also play the role of landmarks, in the sense used by Levitt, Lawton, Chelberg, and Nelson (1987), that qualitatively distinguish places. One simple approach to detecting useful configurations of grid cells would draw on recent methods for feature selection with nearest neighbor methods (e.g., Langley & Sage, in press), which use estimates of accuracy obtained through cross validation to direct search through the space of feature combinations.

In summary, we have presented a framework for representing, using, and learning knowledge about places in which evidence grids play a central role. Our approach draws on earlier work for updating these probabilistic summaries, but diverges from previous schemes by storing a set of local grids in a place library, then retrieving and matching them for use in place recognition. Experimental studies adapted from the machine learning literature indicate that this lazy approach improves recognition accuracy with experience, that sensor noise degrades the learning process, and that improving the quality of stored cases can offset this effect. The experiments also revealed that our method scales well to increased numbers of places, and that some of its power comes from the particular similarity metric used in the matching process. However, additional experiments suggested that using historical knowledge of the places just visited can reduce rather than increase accuracy. Many other environmental and system factors remain to be examined, but the basic approach to lazy learning of place knowledge appears promising and suggests many natural extensions.

11. This scheme is somewhat less lazy than the current version, but the reliance on a sophisticated retrieval mechanism would remain.

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Appendix: Derivation of Previous Place Influences

Recall that our goal is to compute $P(C|E,B)$, the probability that the robot is in current place C given that previous place B and evidence grid E. By applying Bayes Rule, we can rewrite the probability in question as

$$P(C|E,B) = \frac{P(E,B|C)P(C)}{P(E,B)}$$

Now we can expand the denominator by marginalizing over C, giving

$$P(C|E,B) = \frac{P(E,B|C)P(C)}{\sum_C [P(E,B|C)P(C)]}$$

If we are willing to assume that E and B are independent given C (as in the naive Bayesian classifier), we obtain

$$P(E,B|C) = P(E|C)P(B|C)$$

Substituting the above equation for $P(E,B|C)$ in our original expression, we have

$$P(C|E,B) = \frac{P(E|C)P(B|C)P(C)}{\sum_C [P(E|C)P(B|C)P(C)]}$$

and replacing $P(E|C)$ with $\frac{P(C|E)P(E)}{P(C)}$ (by Bayes Rule) gives

$$P(C|E,B) = \frac{\frac{P(C|E)P(E)}{P(C)}P(B|C)P(C)}{\sum_C [\frac{P(C|E)P(E)}{P(C)}P(B|C)P(C)]}$$

Cancelling the $P(C)$ terms in the numerator gives the simplified expression

$$P(C|E,B) = \frac{P(C|E)P(E)P(B|C)}{\sum_C [\frac{P(C|E)P(E)}{P(C)}P(B|C)P(C)]}$$

whereas cancelling the $P(C)$ terms in the denominator gives

$$P(C|E,B) = \frac{P(C|E)P(E)P(B|C)}{\sum_C [P(C|E)P(E)P(B|C)]}$$

Since $P(E)$ is independent of C, we can move $P(E)$ out of the sum in the denominator, producing

$$P(C|E,B) = \frac{P(C|E)P(E)P(B|C)}{P(E)\sum_C [P(C|E)P(B|C)]}$$

Finally, we can cancel the $P(E)$ terms in the numerator and denominator to obtain

$$P(C|E,B) = \frac{P(C|E)P(B|C)}{\sum_C [P(C|E)P(B|C)]}$$

which can be expanded in a number of ways, as described in Section 4.